Workshop on Reconstruction Schemes for MR data
17th August, 2016

Multi-frame Super Resolution based on Sparse Coding

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Outline

1. Sparse coding: Introduction and Mathematical Formulation

2.Super Resolution via Sparse Representation: An overview

3. Our algorithm

4.Results

5. Final comments

Sparse Representation Theory, (Donoho-Candes, 2006)

- Sparse coding consist of writing a signal x as:

$$x = \alpha_1 D_1 + \alpha_2 D_2 + \ \dots + \alpha_n D_n$$

where α is a coefficient vector and D is a matrix whose columns are called *atoms of a dictionary*.

- The idea of expressing a signal in a compact form is an old strategy in Digital Signal processing.
  It is convenient because α exhibits desired properties of the signal.

- Traditionally, D is taken as an orthonormal basis (Fourier basis, Wavelets basis, DCT basis).
  It is convenient because: $\alpha = D^T x$

Where is the novel idea of Sparse Coding?

- The set $D_1, \ldots, D_n$ is **not** a mathematical basis.

- The dictionary D is an overcomplete set that should guarantee the sparseness of α.

- The existence of such D for some signals is a fact statistically proved. For example:
  Image patches can be well-represented as a sparse linear combination of elements taken from a finite and not too big bag.

- Advantage over orthonormal basis:
  Most of the coefficients in α are zero if D is properly selected.

Basic Formulation of Sparse Coding

- We assume any signal of our interest has a sparsity far less the total number of atoms in D.

- First formulation:

$$\min_{\alpha} \|\alpha\|_0 \quad s.t. \quad x = D\alpha$$

Main inconvenience: It is a intractable combinatorial problem.

- Second formulation (Lasso model):

$$\min_{\alpha} \; \|x - D\alpha\|_2 + \lambda \, \|\alpha\|_1$$

A variety of optimization methods have been proposed:
LARS, Coordinate descent, Feature-sign search algorithm

Dictionary Learning

- The calculation of a good overcomplete dictionary is a challenging problem that has evolved a lot in the last years.

- The Dictionary D is usually learned from a set of training examples X:

$$D = arg \min_{D,Z} \|X - DZ\|_2^2 + \lambda \|Z\|_1$$

The problem is not convex in the two variables.

But, it is convex in one of them with the other fixed.

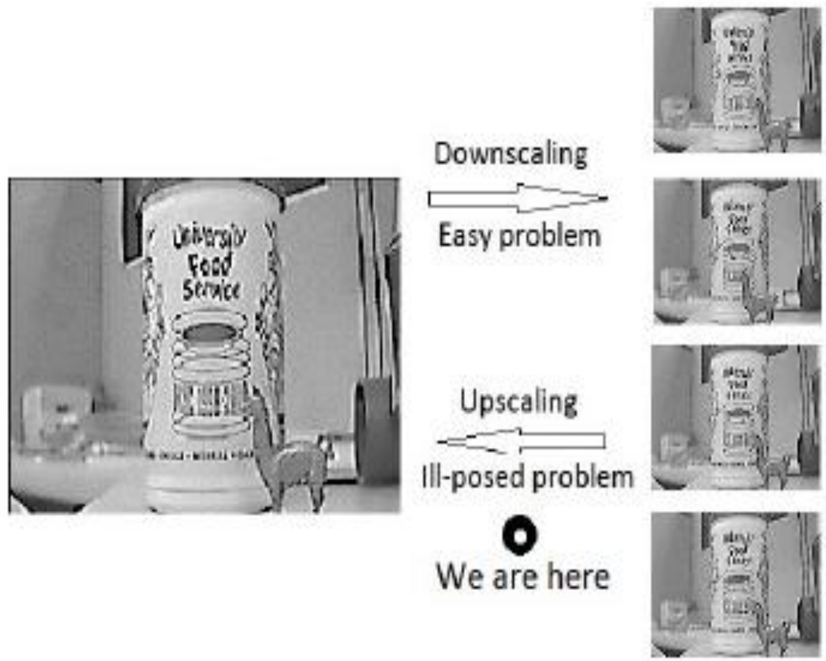
If D is fixed $\longrightarrow$ Nonnegative quadratic linear programming

If Z is fixed $\longrightarrow$ LASSO model

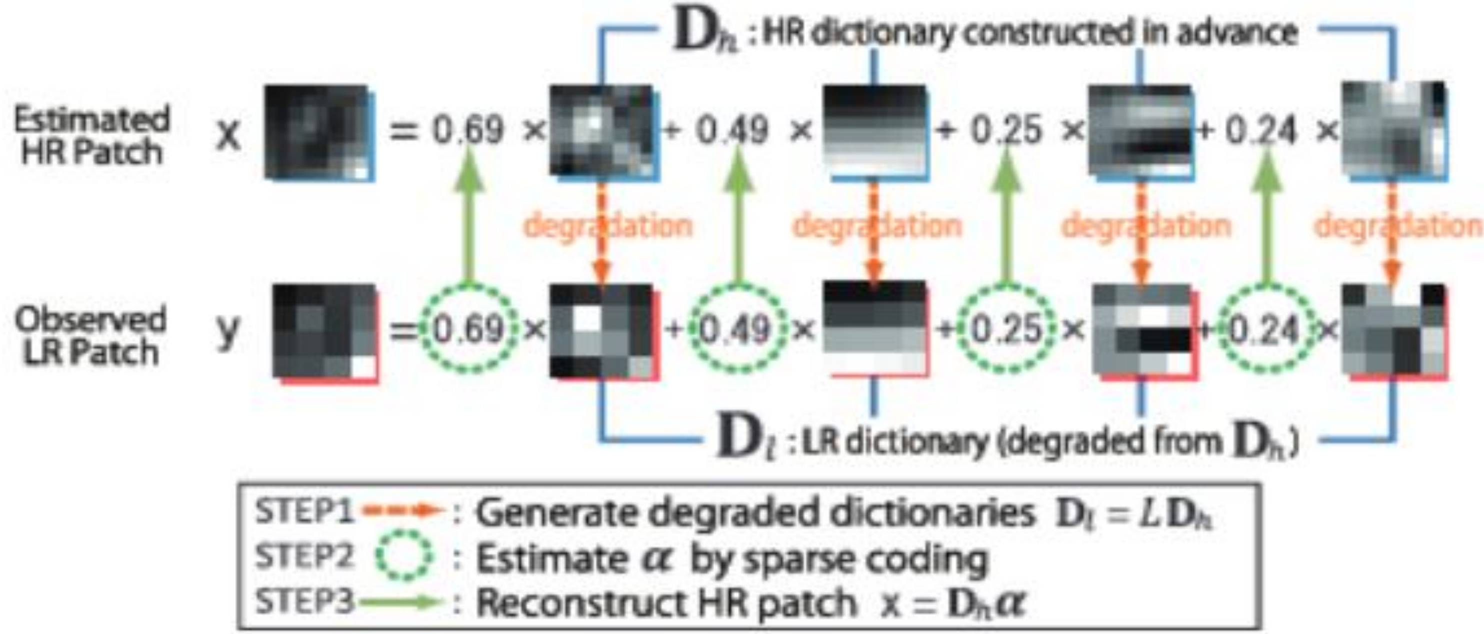
- The learning procedure depends on the applications.

Multi-frame Super-resolution problem: Definition

Create a clear image from low resolution LR images



Super-resolution via Sparse Representation, Yang et al. 2011



• They assume each high resolution patch and the corresponding low resolution patch **share the same sparse linear coefficients** assuming the HR and LR Dictionaries are defined properly.

Super-resolution via Sparse Representation, Yang et al. 2011

- The algorithm has two phases:

  Learning phase:

  Construction of the bilateral dictionaries Dh and Dl

  Testing phase:

  Calculation of the sparse representation coefficients α of each LR patch

  Reconstruction of the HR patch using the coefficients α

- An additional step: It consists of a back projection strategy to diminish the discrepancy produced by noise and blur in the LR image.

Super-resolution via Sparse Representation, Yang et al. 2011

Learning phase: Construction of the bilateral dictionaries Dh and Dl

- Form the training samples

X_h: Set of N high resolution sampled patches	+	Y_l: Set of M low resolution sampled patches

- Calculate Dh and Dl solving the minimization problem

$$\min_{D_h, D_l, Z} \|X_c - D_c Z\|_2^2 + \lambda \|Z\|_1$$

where

$$X_c = \begin{bmatrix} \frac{1}{\sqrt{N}} X_h \\ \frac{1}{\sqrt{M}} Y_l \end{bmatrix} \quad , \quad D_c = \begin{bmatrix} \frac{1}{\sqrt{N}} D_h \\ \frac{1}{\sqrt{M}} D_l \end{bmatrix}$$

Super-resolution via Sparse Representation, Yang et al. ,2011

Testing phase: Reconstruction of the HR image

•Calculate the coefficients α of the sparse representation of each LR patch *y* solving the minimization problem:

$$\alpha^l = \arg\min_{\alpha} \|y - D_l\alpha\|_2^2 + \lambda\|\alpha\|_1$$

•The HR patch x is taken as:

$$x = \alpha^l\, Dh$$

Super-resolution via Sparse Representation, Kato, et al., 2015

- The strategy is an extension of the algorithm proposed by Yang et al. to the case of **multi-frame** case (videos).
- The main idea is to incorporate into the model the subpixel differences between the target frame and the neighbor frames.
- **Learning phase**:
  The training set X is formed by **only** HR patches
  The **unilateral** dictionary Dh is calculated solving the problem

$$\min_{D_h,\alpha}\{\|X - D_h\alpha\|_2 + \delta\|\alpha\|_1\}$$

The LR dictionary Dl is generated by atoms in Dh according to the assumed blur, down-sampling and estimated translation.

Super-resolution via Sparse Representation, Kato, et al., 2015

•**Testing phase**: Reconstruction of the HR image

Calculate the coefficients α of the sparse representation of each LR patch *y* solving the minimization problem:

$$\overline{\alpha} = arg\min_{\alpha} \sum_{j=1}^{K} \|\widetilde{D}^l \alpha - \widetilde{y}\|_2 + \lambda \|\alpha\|_1$$

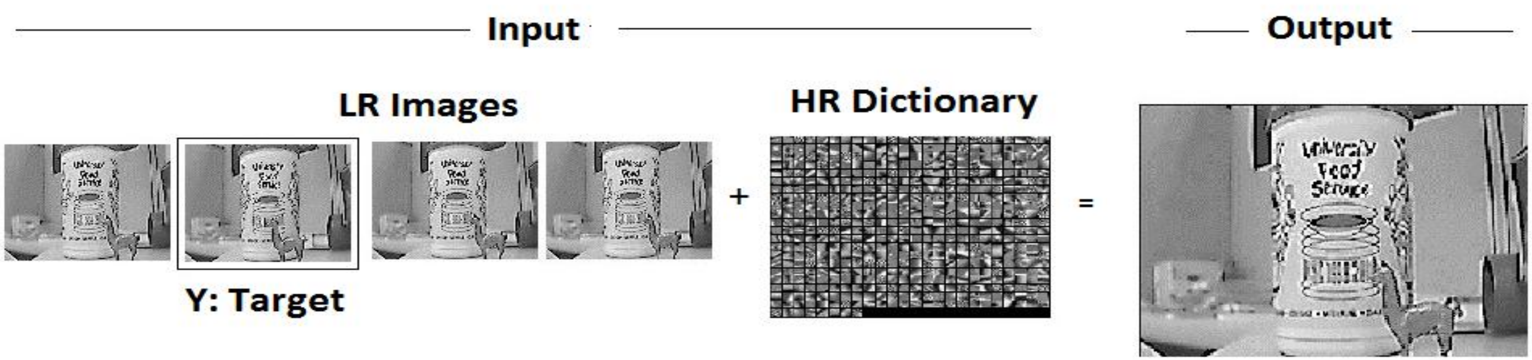
$$\widetilde{y} = \begin{bmatrix} y_1 \\ \\ y_K \end{bmatrix} \qquad \widetilde{D}^l = \begin{bmatrix} DBW_1 D^h \\ \\ DBW_K D^h \end{bmatrix}$$

where D, B and W_i are the down-sampling, blur and motion estimation operator.

The HR patch x is taken as $x = D^h * \overline{\alpha}$

Our algorithm:

- It is a **multi-frame + Unilateral Dictionary strategy**.
- It is basically the algorithm proposed by Kato.
- We include an additional step: a previous bicubic interpolation.



Step 1:
(Patch by patch)
Registration:
Calculate motion estimation by Block Matching

Step 2:
(Patch by patch)
a)Construct the LR Dictionary.
b)Perform the Sparse Coding (calculate α)
c)Reconstruct the patch:

$$x = D * \alpha$$

a)Form X_0 using the patches x

Step 3:
Enforce global reconstruction constrain by back projection:

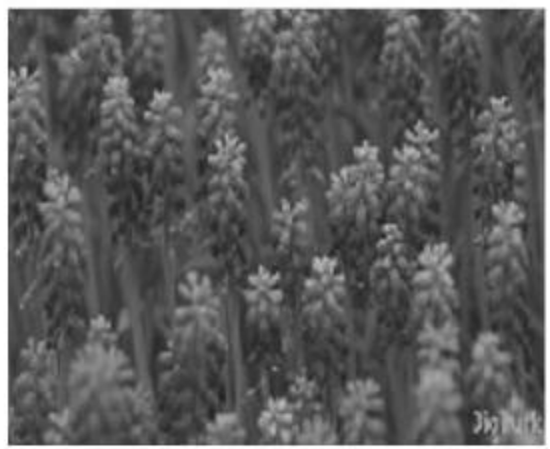
$$\overline{X} = \arg\min_{X} \|DBX - Y\|_2^2 + c\|X - X_0\|_2^2$$

Experiments

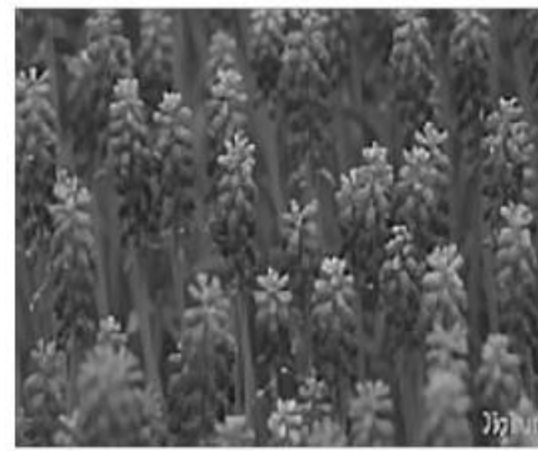
- Lower resolution size: 120 x160 , Scale factor: 2
- Matlab implementation
- Block matching: Sub-pixel algorithm proposed by Shimizu et al., 2006

Original HR Scene

Bicubic Interpolation



Proposed algorithm



Final Comments

- A C++ implementation of the algorithm is in progress. The experimental results will be important to evaluate the proposed algorithm.

- Sparse coding is a promising way to improve the present multi-frame super resolution algorithms.

- Existing super resolution algorithms typically run offline. There is no real possibility for online processing. It is a big challenge.